Sound Waves

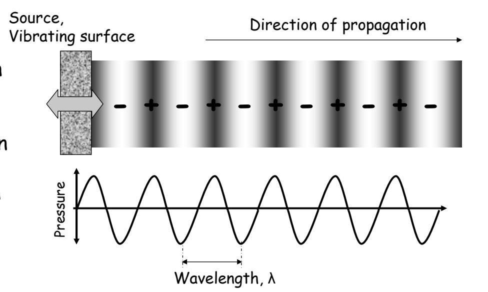
In this lecture

- Sound waves
- Speed of sound
- Acoustic Pressure
- Acoustic Impedance
- Decibel Scale
- Reflection of sound waves
- Doppler effect

Sound Waves

(Longitudinal Waves)

Propagation of zones of alternating compression and rarefaction



Propagation Speed = number of cycles per second X wavelength

$c = f\lambda$

Sound

Range	Frequency
Audible Range	15 – 20,000Hz
Child's hearing	15 – 40,000Hz
Male voice	100 – 1500Hz
Female voice	150 – 2500Hz
Middle C	262Hz
Concert A	440Hz
Bat sounds	50,000 – 200,000Hz
Medical US	2.5 - 40 MHz
Max sound freq.	600 MHz

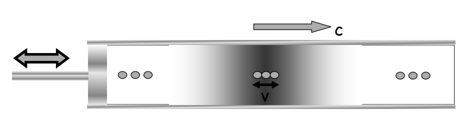
Speed of Sound

$$c = \sqrt{\frac{B}{\rho}}$$

- Speed at which longitudinal displacement of particles propagates through medium
- Speed governed by mechanical properties of medium
- Stiffer materials have a greater Bulk modulus and therefore a higher speed of sound

Sound Particle Velocity

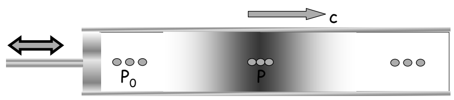
- Velocity, *v*, of the particles in the material as they oscillate to and fro



- Typically several tens of mms^{-1}

Acoustic Pressure

- Pressure, p, caused by the pressure changes induced in the material by the sound energy



- p= $P-P_0$, (where P_0 is normal pressure)
- Typically several tens of kPa

Acoustic Impedance

- Pressure, p, is applied to a molecule it will exert pressure the adjacent molecule, which exerts pressure on its adjacent molecule.
- It is this sequence that causes pressure to propagate through medium.

Acoustic Impedance

- Acoustic pressure increases with particle velocity, v, but also depends upon properties of the medium
- Relationship between acoustic pressure and particle velocity is characterised by the **acoustic impedance** of the medium

$$Z = \frac{p}{v}$$

- Units: kg m^{-2} s^{-1} or a **rayl**

Acoustic Impedance

- Acoustic pressure is analogous to electrical resistance:

$$V = I\,R$$

$$p = v\,Z$$

- Z is a constant for a material (resistance, R) that inhibits velocity (current, I) for a given pressure (voltage, V)

Acoustic Impedance

- Acoustic impedance is also related to the elasticity of the medium
- Stiffer bonds between molecules increases the pressure exerted by a molecule moving with velocity v.
- A springy material will have high molecular motion and absorb sound energy in the bonds
  - less energy will be transferred between molecules

$$Z = \frac{B}{v}$$

Acoustic Impedance

- Wave propagation speed depends upon elasticity of medium and density:

$$Z = \rho\,c = \sqrt{B\,\rho}$$

Acoustic Power

- Sound energy is measured in Joules (J)
- Sound Power in Js^{-1} or Watts (W)
- Again analogous to electricity

$$P = pv = v^2 Z$$

Acoustic Intensity

- Acoustic Intensity is measured in $W\ cm^{-2}$
- Instantaneous power passing through a unit area of material
- Typical intensities used for ultrasound imaging are between 0.01 – 1 $mW\ mm^{-2}$

Recap Logarithms

- If $\log_a(b) = c$
- Then c is the power to which you have to raise a, in order to get b.
- Put more simply, $a^c = b$

Decibel Scale

- Comparative sound intensity is measured using decibels
- Logarithmic unit used to describe a ratio

$$dB = 10\log\left(\frac{I_2}{I_1}\right)$$

- Describe very big ratios using modest numbers

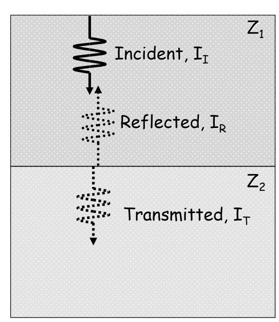
Example

- For an incident ultrasound beam of intensity of 1 Wcm^{-2} is reflected with an intensity of 0.1mW cm^{-2}. Express this power loss in dB.

$$\begin{aligned} dB &= 10\log\left(\frac{I_2}{I_1}\right) \\ &= 10\log\left(\frac{0.0001}{1}\right) \\ &= 10 \times -4 \\ &= -40dB \end{aligned}$$

Reflection & Transmission of Sound Waves

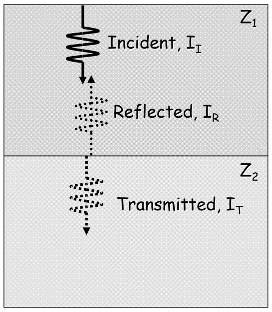
- A pulse of sound incident on an interface between media with different mechanical properties can undergo two processes
- Transmission or Reflection



Reflection & Transmission of Sound Waves

- Amount of reflected and transmitted light depends upon impedance difference

$$I_I = I_T + I_R$$

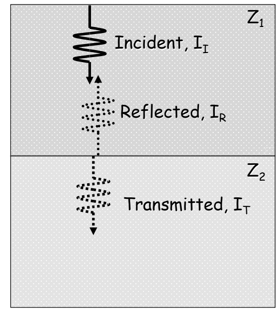


Reflection & Transmission of Sound Waves

- Reflected Intensity

$$I_R = R \times I_I$$

$$R = \frac{I_R}{I_I} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$

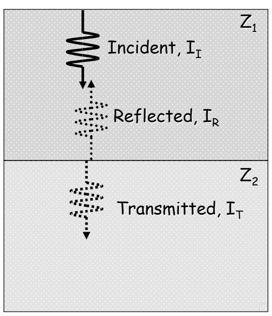


Reflection & Transmission of Sound Waves

- Transmitted Intensity

$$I_T = T \times I_I$$

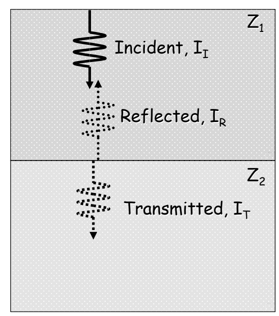
$$T = \frac{I_T}{I_I} = \frac{4Z_1Z_2}{(Z_1 + Z_2)^2}$$



Reflection & Transmission of Sound Waves

- If no energy is lost to medium

$$T + R = 1$$



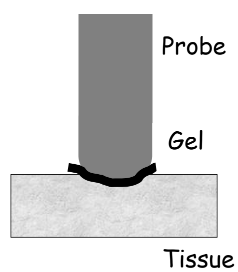
Reflection & Transmission of Sound Waves

- Reflection and transmission of sound waves forms the basis of ultrasound imaging

Impedance Matching

- To optimise Transmission of US into patient from probe an impedance matching medium is used

$$Z_M = \sqrt{Z_T \times Z_p}$$

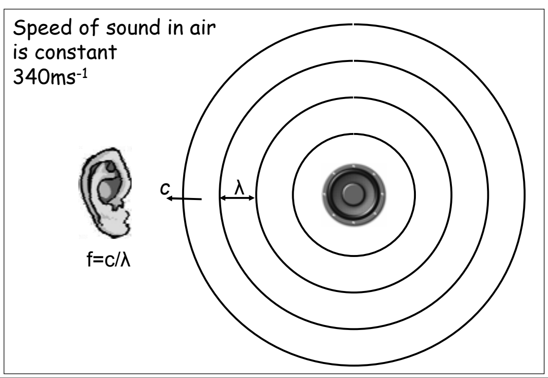


Example

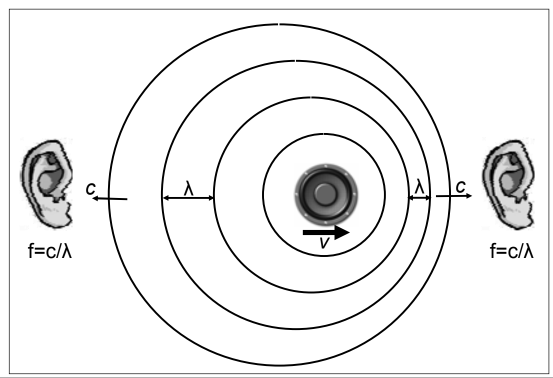
- If a transducer and tissue have acoustic impedances of 30×10^6 & 1.5×10^6 kgm^{-2}s^{-1} respectively, what acoustic impedance should a matching medium have to minimise reflection?

Doppler Effect

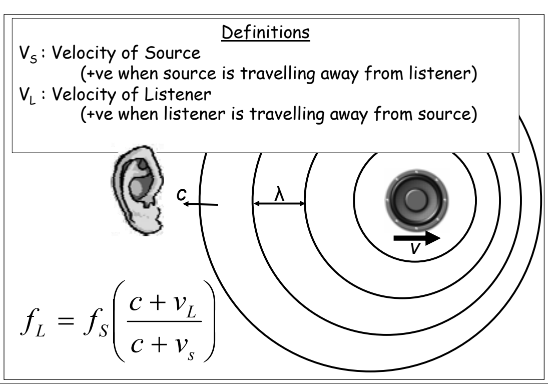
Stationary Sound Source



Moving Sound Source



Moving Sound Source

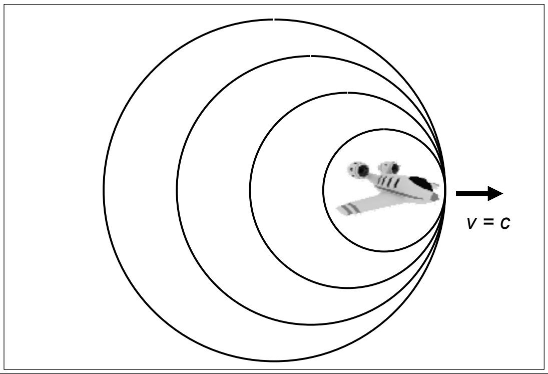


$$f_L = f_S\left(\frac{c+v_L}{c+v_s}\right)$$

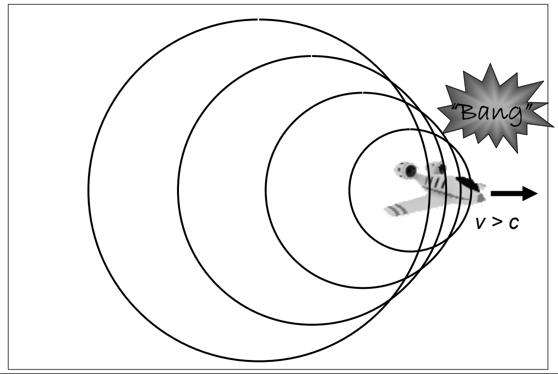
Example

- A police car travelling at 60mph has a siren emitting sound with frequency (f_s) 300Hz. What frequency would a stationary observer measure if the police car was travelling away from her?

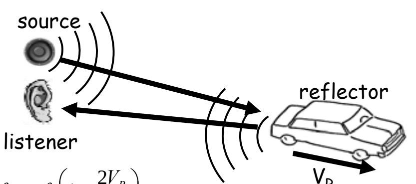
Super-sonic



Super-sonic



Reflected Sound



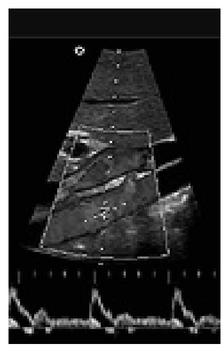
$$f_L = f_S\left(1 - \frac{2V_R}{c}\right)$$

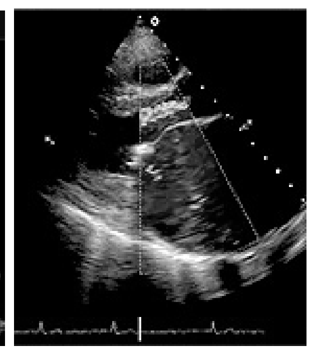
Change in frequency, known as the Doppler Shift, $f_{Doppler}$

$$f_{Doppler} = f_S\left(\frac{2V_R}{c}\right)$$

Remember: If reflector is moving away from transceiver, shift is –ve (i.e.. Red or down shift in freq). If reflector is moving towards transceiver, shift is +ve (i.e. blue or up-shift in freq).

Doppler Ultrasound





Summary

- Sound waves
- Speed of sound
- Acoustic Pressure
- Acoustic Impedance
- Reflection of sound waves
- Decibel Scale
- Doppler effect

Practice Questions

1. A sound wave propagates at 300ms^{-1} through a medium with an acoustic pressure of 10 pa. Calculate the acoustic impedance of the medium
2. A sound wave propagates at 4080ms^{-1} through a medium with a density of 1700 kgm^{-3}. Calculate the acoustic impedance of the medium